Electroweak phase transition in economical 3-3-1 model

Le Hoang Minh

Ho Chi Minh University of Science, HCM National University

VSOP - 22, August, 2016

Overview

Baryogenesis

The baryon asymmetry of the universe (BAU)

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 6 \times 10^{-10}$$

Baryogenesis

The baryon asymmetry of the universe (BAU)

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 6 \times 10^{-10}$$

Sakharov's conditions

1. Baryon number (B) violation.
2. C and CP violation.
3. Departure from thermal equilibrium.

In the context of electroweak baryogenesis:

Third Sakharov condition $\Leftrightarrow$ Condition of first-order electroweak phase transition

Electroweak baryogenesis in SM

Higgs potential in SM

$$V(\varphi) = \mu^2 \varphi^\dagger \varphi + \lambda(\varphi^\dagger \varphi)^2 \tag{1}$$

Electroweak baryogenesis in SM

Higgs potential in SM

$$V(\varphi) = \mu^2 \varphi^\dagger \varphi + \lambda(\varphi^\dagger \varphi)^2 \quad (1)$$

corresponds to the following effective potential

$$V_{eff}(v, T) = \frac{\lambda_T}{4} v^4 + D(T^2 - T_0^2)v^2 - ETv^3 + \Lambda_R \quad (2)$$

$$\lambda_T = \frac{M_H^2}{2v_0^2} + \frac{1}{16\pi^2 v_0^2}\left[3M_Z^4 \ln\left(\frac{M_Z^2}{bT^2}\right) + 6M_W^4 \ln\left(\frac{M_W^2}{bT^2}\right) + \right.$$

$$\left. -12M_t^4 \ln\left(\frac{M_t^2}{b_F T^2}\right)\right]$$

$$E = \frac{1}{12\pi v_0^3}\left(3M_Z^3 + 6M_W^3\right)$$

$$D = \frac{1}{24v_0^2}\left\{3M_Z^2 + 6M_W^2 + \frac{12}{2}M_t^2\right\}$$

$$T_0^2 \equiv \frac{-\mu^2}{2D} = \frac{1}{D}\left\{\frac{M_H^2}{2v_0^2} - \frac{1}{32\pi^2 v_0^2}\left[3M_Z^4 + 6M_W^4 - 12M_t^4\right]\right\}$$

Electroweak baryogenesis in SM

Condition of first-order phase transition

$$\Rightarrow S \equiv \frac{v_c}{T_c} = \frac{2E}{\lambda_{T_c}} \geq 1 \quad (3)$$

For SM case: [Phys.Rev.D71:036001, 2005]

$$\Leftrightarrow m_H \leq 72\ GeV \quad (4)$$

Economical 3-3-1 Model (E331)

In E331 model, there are two Higgs triplets

$$\chi = \begin{pmatrix} \frac{1}{\sqrt{2}}u + G_{X^0} \\ G_{Y^-} \\ \frac{1}{\sqrt{2}}(\omega + H_1^0 + iG_{Z'}) \end{pmatrix}, \quad \phi = \begin{pmatrix} G_{W^+} \\ \frac{1}{\sqrt{2}}(v + H^0 + iG_Z) \\ H_2^+ \end{pmatrix},$$

whose VEVs are, respectively, given by

$$\langle\chi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ 0 \\ \omega \end{pmatrix} \quad ; \quad \langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}$$

These VEVs satisfy the constraint $\omega \gg v \gg u$.

Symmetry breaking in E331 model

$$
\begin{aligned}
E331 &: SU(3)_L \otimes U(1)_X \\
& \qquad \Downarrow \; \omega \\
SM &: SU(2)_L \otimes U(1)_Y \\
& \qquad \Downarrow \; u, v \\
& \qquad U(1)_Q
\end{aligned}
$$

Heavy particles' mass formulations

Names	$m^2(\omega, v)$	Names	$m^2(\omega, v)$
$m^2_{W^\pm}$	$\frac{g^2}{4} v^2$	$m^2_{H^0}$	$\left(2\lambda_2 - \frac{\lambda_3^2}{2\lambda_1}\right) v^2$
$m^2_{Y^\pm}$	$\frac{g^2}{4}(\omega^2 + v^2)$	$m^2_{H_1^0}$	$2\lambda_1\omega^2 + \frac{\lambda_3^2}{2\lambda_1} v^2$
$m^2_{X^0}$	$\frac{g^2}{4}\omega^2$	$m^2_{H_2^\pm}$	$\frac{\lambda_4}{2}(\omega^2 + v^2)$
$m^2_{Z_1} \sim m^2_Z$	$\frac{g^2}{4c_W^2} v^2$		
$m^2_{Z_2} \sim m^2_{Z'}$	$\frac{g^2 c_W^2}{3-4s_W^2}\omega^2$		

My aim

1. Check if this model can have first-order electroweak phase transition.

My aim

1. Check if this model can have first-order electroweak phase transition.
2. Estimate the mass of exotic particles.

My aim

1. Check if this model can have first-order electroweak phase transition.
2. Estimate the mass of exotic particles.
3. Estimate the unknown parameters of the model of exotic particles.

$$
\begin{aligned}
V(\chi, \phi) &= \mu_1^2 \chi^\dagger \chi + \mu_2^2 \phi^\dagger \phi + \lambda_1 (\chi^\dagger \chi)^2 + \lambda_2 (\phi^\dagger \phi)^2 + \\
&+ \lambda_3 (\chi^\dagger \chi)(\phi^\dagger \phi) + \lambda_4 (\chi^\dagger \phi)(\phi^\dagger \chi) \qquad (5)
\end{aligned}
$$

$$\begin{aligned} V(\chi,\phi) &= \mu_1^2\chi^\dagger\chi + \mu_2^2\phi^\dagger\phi + \lambda_1(\chi^\dagger\chi)^2 + \lambda_2(\phi^\dagger\phi)^2 + \\ &+\lambda_3(\chi^\dagger\chi)(\phi^\dagger\phi) + \lambda_4(\chi^\dagger\phi)(\phi^\dagger\chi) \end{aligned} \quad (5)$$

From the Higgs potential, we obtain tree-level V_0:

$$V_0(u,\omega,v) = \frac{\mu_1^2}{2}(u^2+\omega^2) + \frac{\mu_2^2}{2}v^2 + \frac{\lambda_1}{4}(u^4+\omega^4+2u^2\omega^2) \quad (6)$$

$$+\frac{\lambda_2}{4}v^4 + \frac{\lambda_3}{4}(u^2v^2+v^2\omega^2). \quad (7)$$

Due to the constraint $\omega \gg v \gg u$, we can write V_0 as a sum of two parts corresponding to two stages of SSB:

$$V_0(\omega,v) = V_0(\omega) + V_0(v), \quad (8)$$

where $V_0(\omega) = \frac{\mu_1^2}{2}\omega^2 + \frac{\lambda_1}{4}\omega^4$ and $V_0(v) = \frac{\mu_2^2}{2}v^2 + \frac{\lambda_2 v^4}{4}$.The constraint also allow us to decompose $m_b^2(\omega,v) = m_b^2(\omega) + m_b^2(v)$.

Following background field method, we can obtain the high-temperature effective potential for background field ω

$$V_{eff}(\omega) = D'(T^2 - T'^2_0)\omega^2 - E' T\omega^3 + \frac{\lambda'_T}{4}\omega^4, \tag{9}$$

$$\begin{aligned}
D' &= \frac{1}{24\omega_0^2}\left\{n_b m_b^2(\omega_0) + \frac{1}{2}n_q m_q^2(\omega_0)\right\}, \\
T'^2_0 &= \frac{1}{D'}\left\{\frac{1}{4}m^2_{H_1^0}(\omega_0) - \frac{1}{32\pi^2\omega_0^2}\left(n_b m_b^2(\omega_0) - n_q m_q^2(\omega_0)\right)\right\}, \\
E' &= \frac{1}{12\pi\omega_0^3}(n_b m_b^2(\omega_0)), \\
\lambda'_T &= \frac{m^2_{H_1^0}(\omega_0)}{2\omega_0^2}\left\{1 - \frac{1}{8\pi^2\omega_0^2 m^2_{H_1}(\omega_0)}\left[n_b m_b^2(\omega_0)\ln\frac{m_X^2(\omega_0)}{bT^2}\right.\right. \\
&\quad \left.\left. - n_q m_q^2(\omega_0)\ln\frac{m_q^2(\omega_0)}{b_F T^2}\right]\right\},
\end{aligned}$$

where $b = \{Y, X, Z_2, H_1^0, H_2^+\}$; $n_b = \{6, 6, 3, 1, 2\}$; $q = \{U, D_2, D_3\}$; $n_Q = \{12, 12, 12\}$; ω_0 is the value at which the zero-temperature effective potential $V_{eff}^{0^oK}(\omega)$ gets the minimum:

$$V_{eff}^{0^oK}(\omega_0) = 0; \qquad \frac{\partial V_{eff}^{0^oK}(\omega)}{\partial \omega}\bigg|_{\omega=\omega_0} = 0; \qquad \frac{\partial^2 V_{eff}^{0^oK}(\omega)}{\partial \omega^2}\bigg|_{\omega=\omega_0} = m_{H_1^0}^2(\omega)\bigg|_{\omega=\omega_0}.$$

From this conditions, we have the minima of the effective potential: $\omega = 0, \quad \omega \equiv \omega_c = \frac{2E'T_c'}{\lambda'_{T_c'}}$, where ω_c is a critical VEV of χ at the broken state, and T_c' is the critical temperature of phase transition which is given by

$$T_c' = \frac{T_0'}{\sqrt{1 - E'^2/D'\lambda'_{T_c'}}}. \tag{10}$$

Now, we consider the phase transition strength:

$$S' = \frac{\omega_c}{T'_c} = \frac{2E'}{\lambda'_{T'_c}}, \tag{11}$$

which is a function of three unknown masses, $m_{H_1^0}$, $m_{H_2^\pm}$ and $m_Q \equiv m_U = m_{D_2} = m_{D_3}$. For simplicity, we follow the ansatz in [Phys. Rev. D 81, 055015 (2010)] and assume $m_{H_2^\pm} = m_Q$. Then we plot the transition strength S' as the function of $m_{H_1^0}(\omega_c)$ and $m_{H_2^\pm}(\omega_c)$ with ω_c is in the range from 1 TeV to 5 TeV.

Our results show that the heavy particle masses must be in the range of a few TeV, and the strength of the first-order phase transition $SU(3) \rightarrow SU(2)$ is in the range $1 < S' < 13$.

According to [Phys. Rev. D 45, 26852698 (1992)], the accuracy of a high-temperature expansion for the effective potential such as that in Eq. (9) will be better than 5% if $\frac{m_{boson}}{T} < 2.2$, where m_{boson} is the relevant boson mass. This requirement sets the "upper bounds" of the mass ranges of $H_1^0(\omega)$ and $H_2^{\pm}(\omega)$. From Table 16, this requirement is satisfied by all mass ranges of H_1^0, while it narrows slightly most of the mass ranges of $H_2^{\pm}$.

$\omega\,[TeV]$	$T_c'\,[GeV]$	$m_{H_1^0}\,[GeV]$	$m_{H_2^{\pm}}\,[GeV]$	Upper bound [Ge
1	350	$0 < m_{H_1^0} < 300$	$0 < m_{H_2^{\pm}} < 720$	770
2	650	$0 < m_{H_1^0} < 600$	$0 < m_{H_2^{\pm}} < 1440$	1430
3	950	$0 < m_{H_1^0} < 900$	$0 < m_{H_2^{\pm}} < 2150$	2090
4	1300	$0 < m_{H_1^0} < 1200$	$0 < m_{H_2^{\pm}} < 2870$	2860
5	1600	$0 < m_{H_1^0} < 1500$	$0 < m_{H_2^{\pm}} < 3590$	3520

In a similar way, we also obtain the high-temperature effective potential for background field v

$$V_{eff}(v) = D(T^2 - T_0^2)v^2 - ET|v|^3 + \frac{\lambda_T}{4}v^4, \qquad (12)$$

$$D = \frac{1}{24{v_0}^2}\left[n_b m_b^2(\omega_0) - n_q m_q^2(\omega_0)\right],$$

$$T_0^2 = \frac{1}{D}\left\{\frac{m_H^2(v_0) + m_{H_1^0}(v_0)}{4} - \frac{1}{32\pi^2 v_0^2}\left(n_b m_b^2(\omega_0) - n_q m_q^2(\omega_0)\right)\right\},$$

$$E = \frac{1}{12\pi v_0^3}\left(n_b m_b^2(\omega_0) - n_q m_q^2(\omega_0)\right),$$

$$\lambda_T = \frac{m_{H^0}^2(v_0) + m_{H_1^0}^2(v_0)}{2v_0^2}\left\{1 - \frac{1}{8\pi^2 v_0^2(m_{H^0}^2(v_0) + m_{H_1^0}^2(v_0))}\right.$$
$$\left. \times \left[n_b m_b^2(\omega_0)\ln\frac{m_X^2(\omega_0)}{bT^2} - n_q m_q^2(\omega_0)\ln\frac{m_q^2(\omega_0)}{b_F T^2}\right]\right\},$$

where $b = \{W, Y, Z_1, H^0, H_1^0, H_2^+\}$; $n_b = \{6, 6, 3, 1, 1, 2\}$; $q = \{t\}$; $n_Q = \{12\}$; the v_0 is the value at which the zero-temperature effective potential $V_{eff}^{0^o K}(v)$ gets the minimum: From the minimum conditions for $V_{eff}^{0^o K}(v)$

$$
\begin{aligned}
V_{eff}^{0^o K}(v_0) = 0, \quad & \frac{\partial V_{eff}^{0^o K}(v)}{\partial v}\Big|_{v=v_0} = 0, \\
\frac{\partial^2 V_{eff}^{0^o K}(v)}{\partial v^2}\Big|_{v=v_0} = & \left[m^2_{H^0}(v) + m^2_{H_1^0}(v)\right]\Big|_{v=v_0},
\end{aligned} \tag{13}
$$

we have the minima of the effective potential (12):

$$
v = 0, \quad v \equiv v_c = \frac{2ET_c}{\lambda_{T_c}}, \tag{14}
$$

where v_c is the critical VEV of ϕ at the broken state, and T_c is the critical temperature of phase transition which is given by

$$
T_c = \frac{T_0}{\sqrt{1 - E^2/D\lambda_{T_c}}}. \tag{15}
$$

We investigate the phase transition strength

$$S = \frac{v_c}{T_c} = \frac{2E}{\lambda_{T_c}} \tag{16}$$

of this EWPT. To have a first-order phase transition, we requires $S \geq 1$ and plot S as a function of $m_{H_1^0}(v_0)$ and $m_{H_2^\pm}(v_0)$. From the obtained graph, the masses of $H_2^\pm$ and H_1^0 are found to be respectively in the ranges

$$250\,\mathrm{GeV} < m_{H_2^\pm(v)} < 1200\,\mathrm{GeV}$$
$$0\,\mathrm{GeV} < m_{H_1^0(v)} < 620\,\mathrm{GeV},$$

and the transition strength is in the range $1 \leq S < 3$.

If we also considering the requirement, $\frac{m_{boson}}{T} < 2.2$ [Phys. Rev. D 45, 2685-2698 (1992)], then the mass ranges of $H_2^{\pm}$ and H_1^0 are respectively narrowed to:

$$255\,\mathrm{GeV} < m_{H_2^{\pm}} < 280\,\mathrm{GeV},$$
$$0\,\mathrm{GeV} < m_{H_1^0} < 58\,\mathrm{GeV}.$$

Corresponding with these ranges of mass, the range of phase-transition strength is narrowed to $1 \leq S < 1.15$. The critical temperature is about $T = T_c \sim 130\,\mathrm{GeV}$. Thus the EWPT $SU(2) \to U(1)$ is the first-order phase transition, but it seems quite weak.

Combining the obtained results, we have

$$0\,\mathrm{GeV} < m_{H_1^0} = \sqrt{m^2_{H_1^0}(v) + m^2_{H_1^0}(\omega)} < 1501.12\,\mathrm{GeV}, \tag{17}$$

From the phase transition $SU(2) \rightarrow U(1)$, we obtain

$$2.149 < \lambda_4 < 2.591, \qquad 0 < \frac{\lambda_3^2}{2\lambda_1} < 0.0556, \tag{18}$$

For the phase transition $SU(3) \rightarrow SU(2)$, we derived

$$0 < \lambda_4 < 10.3, \qquad 0 < \lambda_1 < 0.45, \tag{19}$$

for any ω. Eqs. (18)-(19) lead to $2.149 < \lambda_4 < 2.591$; $0 < \lambda_1 < 0.45$ and $0 < \frac{\lambda_3^2}{2\lambda_1} < 0.0556$.

Conclusions about phase transition in E331 model

1. $SU(3) \rightarrow SU(2)$ transition is strongly first-order with the strength $1 < S' < 13$.

Conclusions about phase transition in E331 model

1. $SU(3) \to SU(2)$ transition is strongly first-order with the strength $1 < S' < 13$.
2. $SU(2) \to U(1)$ transition is also first-order, but it seems quite weak, $1 \leq S < 1.15$. The critical temperature is about $T = T_c \sim 130\,\mathrm{GeV}$.

Conclusions about phase transition in E331 model

1. $SU(3) \to SU(2)$ transition is strongly first-order with the strength $1 < S' < 13$.
2. $SU(2) \to U(1)$ transition is also first-order, but it seems quite weak, $1 \leq S < 1.15$. The critical temperature is about $T = T_c \sim 130\,\mathrm{GeV}$.
3. $255\,\mathrm{GeV} < m_{H_2^\pm} < 3870\,\mathrm{GeV}$ and $0\,\mathrm{GeV} < m_{H_1^0} < 1501.12\,\mathrm{GeV}$ for $\omega = 5$ TeV lead to $2.149 < \lambda_4 < 2.591$; $0 < \lambda_1 < 0.45$; $0 < \frac{\lambda_3^2}{2\lambda_1} < 0.0556$.

Outlooks

Check three remaining conditions:

- Baryon number violation (Sphaleron rate).
- C and CP violation.
- $\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 6 \times 10^{-10}$

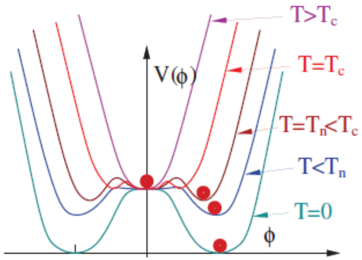
My current work: derive the effective potential in R_ξ-gauge.

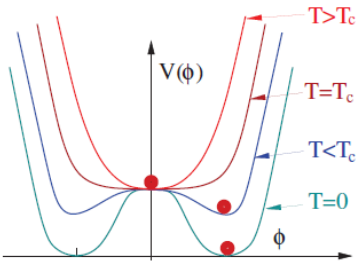
Thank you so much for listening!

This talk is based on Eur. Phys. J. C (2015) 75:342.
With special thanks to Dr. Vo Quoc Phong for introducing me to this wonderful research topic.

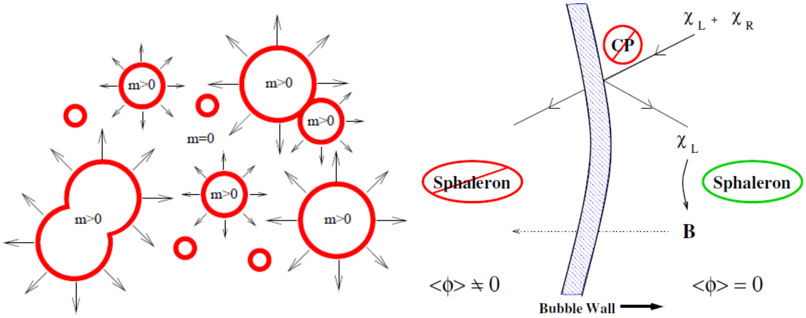
‘

Backup





Backup


m>0
m>0
m>0
m=0
m>0
m>0
m>0
Sphaleron
CP
$\chi_L + \chi_R$
χ_L
Sphaleron
B
$<\phi> \neq 0$
$<\phi> = 0$
Bubble Wall